

October 2021 Shadow Set 1

**1.** Kelly throws a tetrahedral die *n* times and records the number on which it lands for

each throw.

She calculates the expected frequency for each number to be 43 if the die was unbiased.

The table below shows three of the frequencies Kelly records but the fourth one is missing.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Number** | 1 | 2 | 3 | 4 |
| **Frequency** | 46 | 33 | 38 | *x* |

(*a*)Show that *x* = 55

**(1)**

Kelly wishes to test, at the 5% level of significance, whether or not there is evidence

that the tetrahedral die is unbiased.

(*b*)Explain why there are 3 degrees of freedom for this test.

**(1)**

(*c*)Stating your hypotheses clearly and the critical value used, carry out the test.

**(5)**

**(Total for Question 1 is 7 marks)**

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**2.** On a weekday, a factory monitors the randomly produced faulty items, at a mean rate of 1.2 per 15 minutes.

(*a*)Show that the probability that on a weekday at least 3 faults produced by the

Machine in a 60 minute slot is 0.857 to 3 decimal places.

**(2)**

(*b*)Calculate the probability that at least 3 faults are recorded in fewer than

4 out of 7 randomly selected, non‑overlapping 60‑minute periods on a weekday.

**(2)**

The manager of the factory randomly selects 120 non‑overlapping 60‑minute periods

on weekdays.

She records the number of faults produced in each of these 60‑minute periods.

(*c*)Using a Poisson approximation show that the probability of the manager finding at

least 4 of these 60‑minute periods when exactly 10 faults are found is 0.103 to 3 significant figures.

**(4)**

(*d*)Explain why the Poisson approximation may be reasonable in this case.

**(1)**

The manager of the factory decided to investigate if the number of faults produced is different on a Saturday to a weekday. She selects a Saturday at random and records the number faults produced in the first 3.5 hours.

(*e*)Write down the hypotheses for this test.

**(1)**

The manager found that there had been 23 faults in the first 3.5 hours

(*f*)Carry out the test using a 5% level of significance.

**(4)**

**(Total for Question 2 is 14 marks)**

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**3.** A food delivery company delivers takeaway food**.** A random variable *X* represents the number of food deliveries made each day by the company where *X* ~ B (300, 0.72)

A random sample *X*1, *X*2, ... *X*100 is taken.

Estimate the probability that the mean number of food deliveries delivered each day by the

company is greater than 217

**(4)**

**(Total for Question 3 is 4 marks)**

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**4.** A spinner with numbers 0 to 5 is used to play at a games club has the following probability distribution:.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *n* | 0 | 1 | 2 | 3 | 4 | 5 |
| P(*N* = *n*) | 0.1 | *a* | 0.25 | *b* | *c* | 0.2 |

Given that E(2*N* + 5) = 9.6 and P(*N* = 1|*N* < 3) = 

The probability of landing on a 3 is twice the probability of landing on a 4.

(*a*)show that Var(*N*) = 2.71

**(6)**

The games club pay 20p if you land on 3, 4 or 5 otherwise you win 10p.

(*b*)calculate the expected winnings per player.

**(3)**

Bai suggests that, as the mean and variance are close, a Poisson distribution could be

used to model the amount won by a player.

(*c*)State a limitation of the Poisson distribution in this case.

**(1)**

**(Total for Question 4 is 10 marks)**

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**5.** Three bags labelled **A, B** and **C** respectively contain a large number of marbles coloured either yellow or green.

A Marble is drawn from a bag one at a time, the colour is noted and returned to the bag.

The probability of drawing a yellow marble in one go from bag **A** is 0.04

(*a*)Find the probability of drawing at least 3 green marbles before getting a red from bag **A**

**(2)**

(*b*)Find the probability of drawing a second yellow marble from bag **A** on the 10th draw.

**(2)**

The probability of getting a yellow marble on any one draw from bag **B** is *p*.

A marble is drawn and replaced from bag **B** until *n* yellow marbles are drawn. The random variable *D* is the number of marbles drawn.

Given that the mean and the standard deviation of *D* are 22500 and 300 respectively,

(*c*)find the value of *p*.

**(4)**

Tom believe bag **C** has a smaller proportion of yellow marbles than bag **A.** To test this Tom draws marbles from bag **C** until he gets a yellow. Tom defines the random variable *J* to be the number of marbles drawn up to and including the first yellow counter.

(*d*)Stating your hypotheses clearly and using a 5% level of significance, find the

critical region for this test.

**(5)**

Tom gets a yellow marble for the first time on his 24th draw.

(*e*)Giving a reason for your answer, state whether or not there is evidence that bag **C** contains a smaller proportion of yellow marbles than bag **A**.

**(2)**

Given that the probability of getting a yellow from bag **C** on any one draw is 0.013

(*f*)show that the power of the test is 0.380 to 3 significant figures.

**(3)**

**(Total for Question 5 is 18 marks)**

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**6.** The probability generating function of the random variable *X* is

G*X*(*t*) = *k*(2 + *t*)4

where *k* is a constant.

(*a*)Show that *k* =

**(2)**

(*b*)Find P(*X* = 2)

**(2)**

(*c*)Find the probability generating function of *W* = 3*X* + 2

**(2)**

The probability generating function of the random variable *Y* is



Given that *X* and *Y* are independent,

(*d*)find the probability generating function of *U* = *X* + *Y* in its simplest form.

**(2)**

(*e*)Use calculus to find the value of Var(*U*)

**(6)**

**(Total for Question 6 is 14 marks)**

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**7.** A manufacturer has a machine that produces jars of jam.

The volume of jam produced by the machine is normally distributed with

unknown mean *μ* and standard deviation 0.15

Andrew believes that the machine is not working properly and the mean volume of the

jars of jam has decreased.

He takes a random sample of size *n* to test, at the 1% level of significance, the hypotheses

H0: *μ* = 22 H1: *μ* < 22

(*a*)Write down the size of this test.

**(1)**

Given that the actual value of *μ* is 21.8

(*b*)(i) calculate the minimum value of *n* such that the probability of a Type II error is

less than 0.05

Show your working clearly.

**(6)**

(ii) Andrew uses the same sample size, *n*, but now carries out the test at a 5% level of

significance. Without doing any further calculations, state how this would affect

the probability of a Type II error.

**(1)**

**(Total for Question 7 is 8 marks)**

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**TOTAL FOR PAPER IS 75 MARKS**