GCE A Level Further Mathematics (9FM0) – Shadow Paper (Set 1)
9FM0-3B Further Statistics 1

June 2022 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper, which was published in August 2022.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1(a)</td><td>$r = P(X = 0) \times 200$ or $r = P(X = 5) \times 200$ or $s = P(X = 2) \times 200$ or $s = P(X = 3) \times 200$
$r = \underline{\mathbf{6.25}}$ (value may be in table)
$s = \underline{\mathbf{62.5}}$ (value may be in table)</td><td>M1
A1
A1
(3)</td><td>3.4
1.1b
1.1b</td></tr>
<tr><td>(b)</td><td>H_0: B(5,0.5) is a suitable model (o.e.) Condone B(0.5, 5)
H_1: B(5,0.5) is not a suitable model (o.e.)
<table><tr><td>$\frac{(O_i - E_i)^2}{E_i}$</td><td>3.61</td><td>2.74</td><td>0.1</td><td>1.44</td><td>4.80</td><td>15.21</td></tr><tr><td>$\frac{O_i^2}{E_i}$</td><td>19.26</td><td>15.49</td><td>57.6</td><td>82.94</td><td>11.56</td><td>40.96</td></tr></table>$\sum \frac{(O_i - E_i)^2}{E_i} = 27.9$ <u>or</u> $\sum \frac{O_i^2}{E_i} - N = 227.9 - 200 = 27.9$
$v = 6 - 1 = 5$
CV = 11.070
Significant so there is evidence that the researcher's model is not suitable

(6)</td><td>B1
M1
A1
B1
B1ft
A1</td><td>2.5
1.1b
1.1b
1.1b
1.1b
2.2b</td></tr>
<tr><td colspan="4">Total 9</td></tr>
</table>

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>2(a)</td><td>$[E(X) =]\ 0.1b - 2.05$</td><td>B1
(1)</td><td>1.1b</td></tr>
<tr><td>(b)</td><td>$E\left(X^2\right) = (-7)^2 \times 0.4 + (-2)^2 \times 0.15\ \ \left(+0^2 \times 0.2\right) + 7^2 \times 0.15 + 0.1b^2$
$\left(= 27.55 + 0.1b^2\right)$
$"27.55 + 0.1b^2" - ("0.1b - 2.05")^2 \quad (= 43.8875)$
$0.09b^2 + 0.41b - 20.54 (= 0)$
$b =$ <u>13</u> [since $b > 7$]</td><td>M1
M1
M1
A1
(4)</td><td>1.1b
3.1a
1.1b
2.2a</td></tr>
<tr><td>(c)</td><td>
<table>
<tr><td>X</td><td>–7</td><td>–2</td><td>0</td><td>7</td><td>“13”</td></tr>
<tr><td>X^2</td><td>49</td><td>4</td><td>0</td><td>49</td><td>169</td></tr>
<tr><td>$4 - 5X$</td><td>39</td><td>14</td><td>4</td><td>-31</td><td>-61</td></tr>
<tr><td>$X^2 - 4$</td><td>45</td><td>0</td><td>-4</td><td>45</td><td>165</td></tr>
<tr><td>$-5X$</td><td>35</td><td>10</td><td>0</td><td>-35</td><td>-65</td></tr>
<tr><td>$X^2 + 5X$</td><td>14</td><td>-6</td><td>0</td><td>84</td><td>234</td></tr>
<tr><td>$X^2 + 3X - 4$</td><td>10</td><td>-10</td><td>-4</td><td>80</td><td>230</td></tr>
</table>
$P\left(X^2 < 4 - 5X\right) = P(X = -2) + P(X = 0)$
$= \mathbf{\underline{0.35}}$</td><td>M1
A1ft

M1
A1
(4)</td><td>2.1
1.1b

2.2a
1.1b</td></tr>
<tr><td colspan="4">Total 9</td></tr>
</table>

Qu	Scheme	Marks	AOs
3(a)	$W \sim \text{Po}(15.6)$ and $P(W \geq 21) = 1 - P(W \leq 20)$ or suitable 3sf probs $P(W \geq 21) = 0.110589$ awrt **0.111**	M1 A1 **(2)**	3.4 1.1b
(b)	[S = # calls per day, $S \sim \text{Po}(0.6)$] $P(S > 2) = 0.023115$ awrt $X \sim \text{B}(200, \text{“}0.023115\text{”})$ $Y \sim \text{Po}(\text{“}4.623\ldots\text{”})$ suitable 3sf probs $= 0.02012116\ldots$ awrt **0.0201**	B1 M1 M1 A1 **(4)**	1.1b 3.3 3.4 1.1b
(c)	$H_0: \lambda = 21$ $H_1: \lambda < 21$ $U \sim \text{Po}(21)$ $P(U \leq 11) = 0.01290476$ [0.013 < 0.05 or there is sufficient evidence to reject H_0] There is sufficient evidence at the 5% level of significance that the number of calls received per day is lower in winter **or** rate of calls is lower in winter **or** less calls per day in winter (o.e.)	B1 B1 M1 A1 **(4)**	2.5 3.3 1.1b 2.2b
(d)	$C \sim \text{Po}(0.6 \times n + 0.3 \times n)$ [= Po(0.9n)] or $D \sim \text{B}(n, e^{-0.9}$ or awrt 0.407) $e^{-0.9} < 0.002$ or $-0.9n < \ln(0.002)$ or $n > 6.9\ldots$ $n = \underline{7}$	M1 M1 A1 **(3)**	3.1b 1.1b 1.1b
(e)	The rate of calls per day is constant **or** the number of calls occurring in non-overlapping time intervals is independent. **or** number of calls per day is independent (o.e.)	B1 **(1)**	2.4
			Total 14

Question	Scheme	Marks	AOs
4(a) (i)	$[W \sim \text{Geo}(0.22)]$ $P(W = 5) = 0.78^5 \times 0.22$ =	M1	3.3
	$0.081433\ldots$ awrt **0.0814**	A1	1.1b
		(2)	
(ii)	$P(W \leq 4) = 1 - 0.78^4 = 0.629849\ldots$	M1	3.1b
	awrt **0.630**	A1	1.1b
		(2)	
(iii)	$X \sim \text{B}(5, 0.22)$	M1	3.3
	$P(X = 3) = 0.0647824\ldots$ awrt **0.0648**	A1	1.1b
		(2)	
(iv)	$[Y \sim \text{NB}(3, 0.22)]$ using a neg bin	M1	3.3
	$P(Y \leq 5) = P(Y = 3) + P(Y = 4) + P(Y = 5)$	M1	3.1b
	$= 0.22^3 + \binom{3}{2} \times 0.22^3 \times 0.78 + \binom{4}{2} \times 0.22^3 \times 0.78^2$	M1	3.4
	$= 0.07443377$ awrt **0.0744**	A1	1.1b
		(4)	
(b)	$P(\text{Zac wins})$	M1	3.1b
	$= \frac{0.78 \times 0.22}{1 - 0.78^2}$ oe	M1	1.1b
	$= 0.4382\ldots = 0.438$*	A1cso*	2.1
		(3)	
			Total 13

Question	Scheme	Marks	AOs
5	Geo (0.4) $\mu = \frac{1}{0.4}$ [or exact equivalent]	B1	1.1b
	$\sigma^2 = \frac{1-0.4}{0.4^2}$ [or exact equivalent]	B1	1.1b
	CLT $\Rightarrow \overline{X} \approx \text{N}\left(\frac{10}{4}, ...\right)$ oe	M1	2.1
	$\Rightarrow \overline{X} \approx \text{N}\left(\frac{10}{4}, \frac{3}{200}\right)$ and attempt (sight of) $P\left(\overline{X} < 2.65\right)$	M1	3.4
	$= 0.88966\ldots$ awrt **0.890**	A1	1.1b
		(5)	
			Total 5

Question	Scheme	Marks	AOs
6(a)	$G_V(t)=\frac{36}{121}t^4+\frac{60}{121}t^5+\frac{25}{121}t^6$ or $G_V(t)=t^4\left(\frac{36}{121}+\frac{60}{121}t+\frac{25}{121}t^2\right)$	M1	1.1b
	$G_V(t)=t^4\left(\frac{5}{11}t+\frac{6}{11}\right)^2$ *	A1* cso	2.1
		(2)	
(b)(i)	$G_W'(t)=5t\left(\frac{5}{11}t+\frac{6}{11}\right)^{10}+\left(\frac{5}{11}t+\frac{6}{11}\right)^{11}$	M1	2.1
	$\left[G_W'(1)=\right]$ **6**	A1	1.1b
(ii)	$G_W''(t)=5\left(\frac{5}{11}t+\frac{6}{11}\right)^{10}+\frac{250}{11}t\left(\frac{5}{11}t+\frac{6}{11}\right)^{9}+5\left(\frac{5}{11}t+\frac{6}{11}\right)^{10}$ oe	M1	2.1
	$G_W''(1)=\frac{360}{11}$	A1	1.1b
	$\text{Var}(W)=\text{"}\frac{360}{11}\text{"}+\text{"6"}-(\text{"6"})^2$	M1	2.1
	$=\frac{30}{11}$	A1	1.1b
		(6)	
(c)	$G_X(t)=t^4\left(\frac{5}{11}t+\frac{6}{11}\right)^2\times t\left(\frac{5}{11}t+\frac{6}{11}\right)^{11}$	M1	3.1a
	$=t^5\left(\frac{5}{11}t+\frac{6}{11}\right)^{13}$	A1	1.1b
		(2)	
(d)	$G_Y(t)=t^5\times\left(t^4\right)^5\times\left(\frac{5}{11}t^4+\frac{6}{11}\right)^{13}$	M1	3.1a
	$=t^{25}\left(\frac{5}{11}t^4+\frac{6}{11}\right)^{13}$	A1	1.1b
		(2)	
(e)	$P(Y=33)$ is coefficient of t^{33} ie t^{33} ie $\ldots+t^{25}\times{}^{13}C_2\left(\frac{5}{11}t^4\right)^2\left(\frac{6}{11}\right)^{11}+\ldots$ or $P(X=7)$ need coefficient of t^7 ie $\ldots+t^5\times{}^{13}C_2\left(\frac{5}{11}t\right)^2\left(\frac{6}{11}\right)^{11}+\ldots$	M1	1.1b
	$[P(Y=33)=]$ 0.0204924…	A1	1.1b
		(2)	
			(14 marks)

Question	Scheme	Marks	Aos
7(a)	$\overline{X} \sim \mathrm{N}(900, 80)$ (May be implied by correct prob or z value seen) $P(\overline{X} > 920) = 0.01267365...$ 0.01267365… or $z = 8.944$ 0.01267365… < 0.05 or $z = 2.236067\ldots > 1.6449$ therefore reject H_0. There is evidence that the mean weight of the flour in a bag is not 900 g **or** evidence of a change in mean weight of flour in a bag	M1 A1 M1 A1 cso **(4)**	3.3 3.4 1.1b 2.2b
(b)	$\left[\overline{Y} \sim \mathrm{N}\left(900, \frac{1600}{n}\right) \Rightarrow\right] \quad \frac{c-900}{\left(\frac{40}{\sqrt{n}}\right)} = 1.2816$ $c = 900 + \frac{51.264}{\sqrt{n}}$	M1 A1 **(2)**	3.4 1.1b
(c)	$\frac{900 + \frac{51.264}{\sqrt{n}} - 920}{\frac{40}{\sqrt{n}}} = -3.0902$ $174.872 = 40\sqrt{n}$ $n =$ **76** $c = 905.86\ldots$	M1 A1ft dM1 A1 A1 **(5)**	3.4 1.1b 1.1b 2.1 1.1b
			(11 marks)