

**June 2022, Shadow Set 1**

**1.** A researcher is investigating the number of female cubs present in litters of size 5

He believes that the number of female cubs in a litter can be modelled by B(5, 0.5)

He randomly selects 200 litters each of size 5 and records the number of female cubs.

The results are recorded in the table below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of female cubs | 0 | 1 | 2 | 3 | 4 | 5 |
| Observed number of litters | 11 | 22 | 60 | 72 | 19 | 16 |

He calculated the expected frequencies as follows

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of female cubs | 0 | 1 | 2 | 3 | 4 | 5 |
| Expected number of litters | *r* | 31.25 | *s* | *s* | 31.25 | *r* |

(*a*)Find the value of *r* and the value of *s*

**(3)**

(*b*)Carry out a suitable test, at the 5% level of significance, to determine whether or not

the number of female cubs in a litter can be modelled by B(5, 0.5)

You should clearly state your hypotheses and the critical value used.

**(6)**

**(Total for Question 1 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**2.** The discrete random variable *X* has probability distribution

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | –7 | –2 | 0 | 7 | *b* |
| P(*X* = *x*) | 0.4 | 0.15 | 0.2 | 0.15 | 0.1 |

where *b* is a constant and *b* > 7

(*a*)Find E(*X* ) in terms of *b*

**(1)**

Given that Var(*X* ) = 43.8875

(*b*)find the value of *b*

**(4)**

(*c*)Find P(*X*2 < 4 – 5*X* )

**(4)**

**(Total for Question 2 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**3.** During the summer, mountain rescue team *A* receives calls for help randomly with a rate

of 0.6 per day.

(*a*)Find the probability that during the summer, mountain rescue team *A* receives at

least 21 calls for help in 26 randomly selected days.

**(2)**

The leader of mountain rescue team *A* randomly selects 200 summer days from the

last few years.

She records the number of calls for help received on each of these days.

(*b*)Using a Poisson approximation, estimate the probability of the leader finding at least 10

of these days when more than 2 calls for help was received by mountain rescue team *A*.

**(4)**

Mountain rescue team *A* believes that the number of calls for help per day is lower in

the winter than in the summer. The number of calls for help received in 35 randomly

selected winter days is 11

(*c*)Use a suitable test, at the 5% level of significance, to assess whether or not there is

evidence that the number of calls for help per day is lower in the winter than in the

summer. State your hypotheses clearly.

**(4)**

During the summer, mountain rescue team *B* receives calls for help randomly with a rate

of 0.3 per day, independently of calls to mountain rescue team *A*.

The random variable *C* is the total number of calls for help received by mountain rescue

teams *A* and *B* during a period of *n* days in the summer.

On a Monday in the summer, mountain rescue teams *A* and *B* each receive a call

for help.

Given that over the next *n* days P(*C* = 0) < 0.002

(*d*)calculate the minimum value of *n*

**(3)**

(*e*)Write down an assumption that needs to be made for the model to be appropriate.

**(1)**

**(Total for Question 3 is 14 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4.** In a game a spinner is spun repeatedly. When the spinner is spun, the probability of it

landing on blue is 0.22

(*a*)Find the probability that the spinner lands on blue

(i) for the first time on the 5th spin,

**(2)**

(ii) for the first time before the 5th spin,

**(2)**

(iii) exactly 3 times during the first 5 spins,

**(2)**

(iv) for the 3rd time on or before the 5th spin.

**(4)**

Zac and Izana play the game. They take turns to spin the spinner. The winner is the first

one to have the spinner land on blue. Izana spins the spinner first.

(*b*)Show that the probability of Zac winning is 0.438 to 3 significant figures.

**(3)**

**(Total for Question 4 is 13 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**5.** A random sample of 250 observations is taken from a geometric distribution with

parameter 0.4

Estimate the probability that the mean of the sample is less than 2.65

**(5)**

**(Total for Question 5 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**6.** The discrete random variable *V* has probability distribution

|  |  |  |  |
| --- | --- | --- | --- |
| *v* | 4 | 5 | 6 |
| P(*V* = *v*) |  |  |  |

(*a*)Show that the probability generating function of *V* is

  **(2)**

The discrete random variable *W* has probability generating function



(*b*)Use calculus to find

(i) E(*W*)

**(2)**

(ii) Var(*W*)

**(4)**

Given that *V* and *W* are independent,

(*c*)find the probability generating function of *X* = *V* + *W* in its simplest form.

**(2)**

The discrete random variable *Y* = 4*X* + 5

(*d*)Find the probability generating function of *Y*

**(2)**

(*e*)Find P(*Y* = 33)

**(2)**

**(Total for Question 6 is 14 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.** A machine fills bags with flour. The weight of flour delivered by the machine into a bag,

*X* grams, is normally distributed with mean *μ* grams and standard deviation 40 grams.

To check if there is any change to the mean weight of flour delivered by the machine

into each bag, Olaf takes a random sample of 20 bags. The weight of flour, *x* grams, in

each bag is recorded and ** = 920

(*a*)Test, at the 10% level of significance, H0 : *μ* = 900 against H1 : *μ* ≠ 900

**(4)**

Olaf decides to alter the test so that the hypotheses are H0 : *μ* = 900 and H1 : *μ* > 900 but

keeps the level of significance at 10%

He takes a second sample of size *n* and finds the critical region,**> *c*

(*b*)Find an equation for *c* in terms of *n*

**(2)**

When the true value of *μ* is 920 grams, the probability of making a Type II error

is 0.0010, to 2 significant figures.

(*c*)Calculate the value of *n* and the value of *c*

**(5)**

**(Total for Question 7 is 11 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**TOTAL FOR PAPER IS 75 MARKS**